Sample Size Planning for MLM

PSYC 575

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Week Learning Objectives

- Describe the importance of having sufficient sample size for scientific research
- Describe conceptually the steps for sample size planning: precision analysis and power analysis
- Perform power analysis for MLM using the PowerUpR application and the `simr` package
- Understand the effect of uncertainty in parameter values and explore alternative approaches for sample size planning

Why Sample Size?

Small Sample Size is a Problem Because . . .

Low power

Misleading and noisy results[1]

- When coupled with publication bias (statistical significance filter)[2] [3]

Nonreproducible findings

[1] See Maxwell (2004)

[2] See the graph on this blog post

[3] See also Vasishth et al. (2018)

Review: Sampling distributions

What is the null distribution?

- Suppose we examine the effect of a therapy on eating disorder
- We test against the null hypothesis $H_0 : \gamma_{01} = 0$, where γ_{01} is the fixed effect of the therapy on eating disorder

What is the alternative distribution?

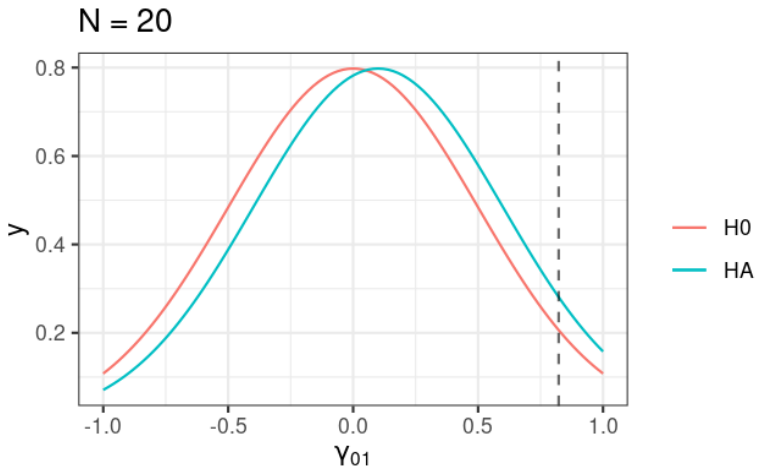
- Assume that the true effect of this therapy is $\gamma_{01} = .1$

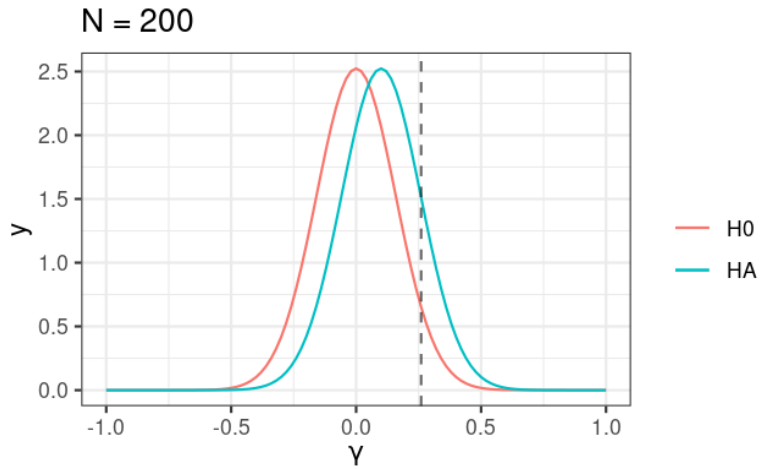
Sampling Distribution as a Function of Sample Size

Assume true effect is $\gamma_{01} = 0.10$

Let's say

- when $N = 20, p < .05$ when $\hat{\gamma} \geq 0.82$
- when $N = 200, p < .05$ when $\hat{\gamma} \geq 0.26$





Steps for Sample Size Planning

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1. Write down your model equations
2. List out all parameters in the model
3. Determine if you want to achieve a desired level of

a. Power, or

b. Precision

Step 1: Write down model equations

Group-based therapy for eating disorder (cluster-randomized trial)

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Level-1

$$Y_{ij} = \beta_{0j} + \beta_{1j} X\_\mathrm{cmc}_{ij} + e_{ij}$$

$$e_{ij} \sim N(0, \sigma)$$

Level-2

$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11} W_j + u_{1j}$$
$$\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_0^2 & \\ \tau_{01} & \tau_1^2 \end{bmatrix} \right)$$

- γ_{10}: X (purely level-1 with ICC = 0)
- γ_{01}: W (level-2)
- γ_{11}: $W \times X$ (cross-level interaction)

Step 2: List out all parameters

1. Fixed effects: $\gamma_{00}, \gamma_{01}, \gamma_{10}, \gamma_{11}$
2. Random effects: $\tau_0^2, \tau_1^2, \tau_{01}$
3. Number of clusters: J
4. Cluster size: n

Level-1

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$$e_{ij} \sim N(0, \sigma)$$

Level-2

$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j}$$
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Standard Error and Precision Analysis

Sample Size and *SE*/Post. *SD*

In the previous graph, when $N = 20$, the sample estimate is likely to be anywhere between -0.4 and 0.6

$$SE \propto \frac{1}{\sqrt{N}}$$

One goal of sample size planning is to

> Have sufficient sample size to get precise (low *SE*) sample estimates of an effect

Analytic Formulas of *SE*

J = Number of clusters; n = Cluster size

- E.g., $J = 100$ schools; $n = 10$ students per school

Assuming $\tau_{01} = 0$

$$SE(\gamma_{01}) = \sqrt{\frac{1}{S_W^2}\left(\frac{\tau_0^2}{J} + \frac{\sigma^2}{Jn}\right)}$$

$$SE(\gamma_{10}) = \sqrt{\frac{\tau_1^2}{J} + \frac{\sigma^2}{JnS_X^2}}$$

$$SE(\gamma_{11}) = \sqrt{\frac{1}{S_W^2}\left(\frac{\tau_1^2}{J} + \frac{\sigma^2}{JnS_X^2}\right)}$$

Precision Analysis

Group-based therapy for eating disorder (cluster-randomized trial)

- Intervention at group level
- 10 participants per group
- Outcome standardized (i.e., *SD* = $\sqrt{\tau_0^2 + \sigma^2} = 1$)
  - γ = Cohen's d
- ICC = .3 (i.e., $\tau_0^2 = .3$)
- Goal: estimate J such that $SE(\gamma_{10}) \leq .1$
  - E.g., if we estimated the sample effect size to be $d = .25$, the 95% CI would be approximately [.05, .45].

Calculating J

When the predictor is binary (e.g., treatment-control), if half of the groups is in one condition, $S^2_W = 0.25$

- Otherwise, if 30% in one condition, $S^2_W = 0.3 \times 0.7$
- $\tau^2_0 = 0.3, \sigma^2 = 0.7, n = 10$

E.g., if $J = 30$

$$SE(\gamma_{01}) = \sqrt{\frac{1}{S^2_W}\left(\frac{\tau^2_0}{J} + \frac{\sigma^2}{Jn}\right)} = \sqrt{\frac{1}{0.25}\left(\frac{0.3}{30} + \frac{0.7}{(30)(10)}\right)} = 0.222$$

Keep trying, and you'll find ...

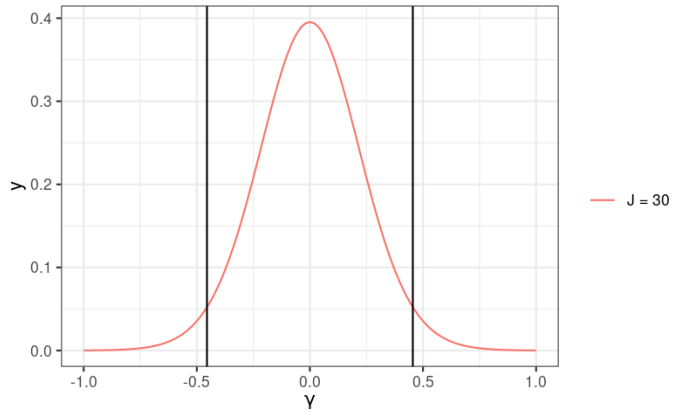
When J = 148, $SE(\gamma_{01}) = 0.1$

So you'll need 148 groups (74 treatment, 74 control)

Power Analysis

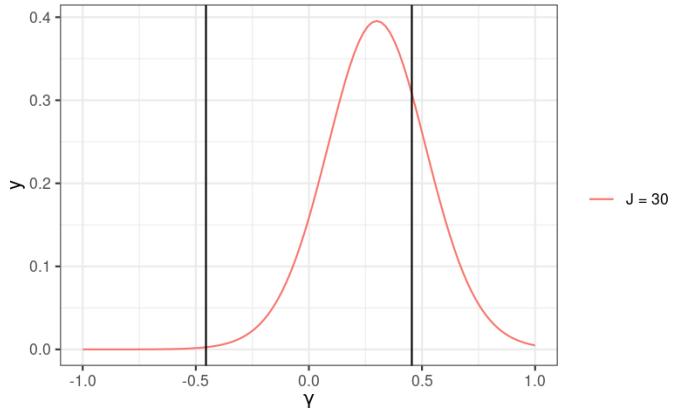
Two-tailed test, $\alpha = .05$

$H_0 : \gamma_{01} = 0$



Critical region: $\hat{\gamma}_{01} \leq -0.45$ or $\hat{\gamma}_{01} \geq 0.45$

$H_1 : \gamma_{01} = 0.3$

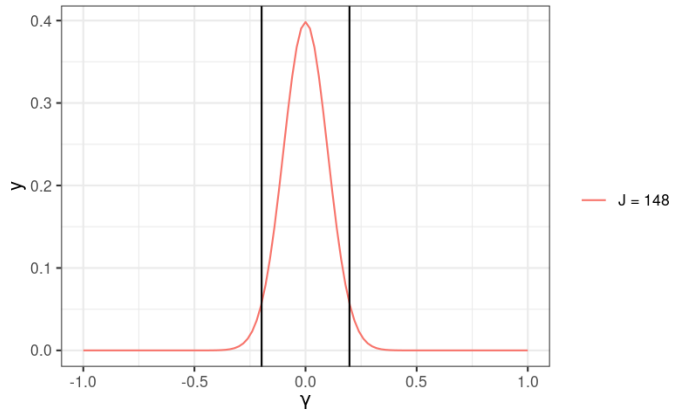


Power[1]
$\approx P(\hat{\gamma}_{01} \leq -0.45) + P(\hat{\gamma}_{01} \geq 0.45) = 0.247$

[1] In practice, we need to incorporate the sampling variability of the standard error as well, so this power calculation is only a rough approximation.

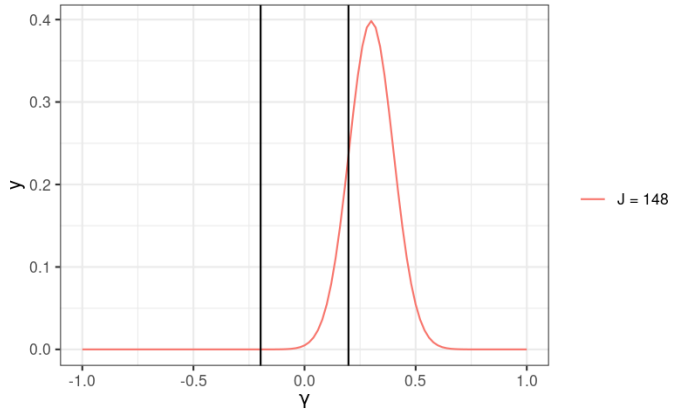
Two-tailed test, $\alpha = .05$

$H_0 : \gamma_{01} = 0$



Critical region: $\hat{\gamma}_{01} \leq -0.2$ or $\hat{\gamma}_{01} \geq 0.2$

$H_1 : \gamma_{01} = 0.3$



Power
$\approx P(\hat{\gamma}_{01} \leq -0.2) + P(\hat{\gamma}_{01} \geq 0.2) = 0.846$

Tools for Power Analysis

1. Stand-alone programs
    - Optimal Design
    - PinT
2. R packages
    - `simr`
3. Spreadsheet/Webapp
    - PowerUp!

See more discussion in Arend & Schäfer (2019)

PowerUpR Shiny App

https://powerupr.shinyapps.io/index/

Monte Carlo Simulation for Power Analysis

- Simulate a large number (e.g., R = 1,000) of data sets based on given effect size, ICC, etc
- Fit an MLM to each simulated data
- Power $\approx$ Proportion of times $p < \alpha$

See sample R code for using `simr`

Uncertainty in Parameter Values

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In the PowerUpR demo, to calculate the number of clusters J need to achieve 80% power, we determined

1. Type I error rate = .05
2. Two tailed test = TRUE
3. `g2`, `r21`, `r22` = 0, as we did not include any covariates
4. `p` = .5, for a balanced design (half treatment, half control)

However, we need to guess the values of

1. Effect size = .3?
2. ICC = .3?

The Effect of Uncertainty in Power

Ignoring uncertainty

- The more uncertainty we have but ignore about a parameter value, the more power loss we will have in our study (red curve)
- Uncertainty in both effect size and ICC can further reduce our power
- The more uncertainty we have, the more samples we need to achieve 80% power

Hybrid Classical-Bayesian approach

- Incorporates uncertainty for sample size planning
- Instead of plugging in a point value of a guess, we can specify how much uncertainty we have (e.g., standard error of γ_{01} from a previous study)

$$\delta \sim N(.3, .1) \quad \rho \sim \text{Beta}(a, b)$$

- where a, b can be calculated by $\hat{\rho} = .3$ and $\sigma_\rho = .1$ (estimate and uncertainty about ρ)

hcbr Shiny App

http://winnie-wy-tse.shinyapps.io/hcb_shiny

Additional Notes on Power

- Increasing J usually leads to higher power than increasing n
- Balanced designs generally have higher power than unbalanced designs
- Larger sample size required for testing level-2 predictors
- Testing an interaction requires a much larger sample size
    - E.g., 16 times larger than for a main effect